Solutions to JEE(Main)-2015

PAPER – 1 CHEMSITRY, MATHEMATICS & PHYSICS

Test Booklet Code

B

Important Instructions:

1. The test is of **3 hours** duration.
2. The Test Booklet consists of **90** questions. The maximum marks are 360.
3. There are ***three*** parts in the question paper A, B, C consisting of **Chemistry, Mathematics and Physics** having 30 questions in each part of equal weightage. Each question is allotted **4 (four)** marks for each correct response.
4. Candidates will be awarded marks as started above in instruction No. 3 for correct response of each question. ¼(one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
5. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 4 above.

PART A – CHEMISTRY

*1. Which of the following is the energy of a possible excited state of hydrogen?

(1) –6.8 eV (2) –3.4 eV
(3) +6.8 eV (4) +13.6 eV

Sol. **2**

$$E_n = -13.6 \times \frac{Z^2}{n^2}\ \text{eV}$$

For hydrogen Z = 1

$$E_n = -13.6 \times \frac{1}{n^2}\ \text{eV}$$

$$E_2 = \frac{-13.6}{4} = -3.4\ \text{eV}$$

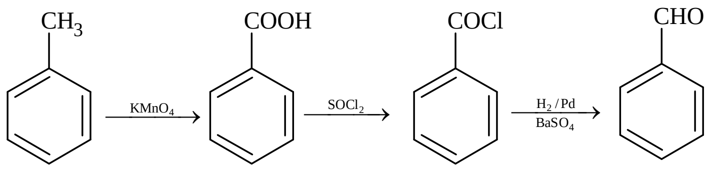
2. In the following sequence of reactions:

$$\text{Toluene} \xrightarrow{KMnO_4} A \xrightarrow{SOCl_2} B \xrightarrow[BaSO_4]{H_2/Pd} C,$$

The product C is :

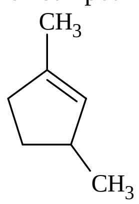
(1) $C_6H_5CH_3$ (2) $C_6H_5CH_2OH$
(3) C_6H_5CHO (4) C_6H_5COOH

Sol. **3**



(A) (B) (C)

*3. Which compound would give 5-keto-2-methyl hexanal upon ozonolysis?

(1)


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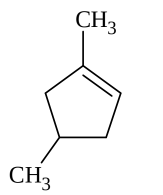

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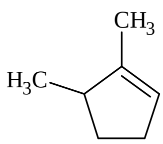

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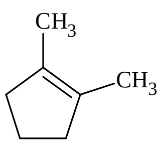
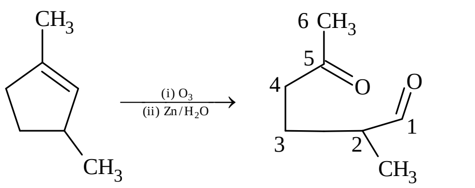


Sol. **1**



5 - keto- 2 - methyl hexanal

*4. The ionic radii (in Å) of N^{3-}, O^{2-} and F^{-} are respectively:
(1) 1.36, 1.71 and 1.40
(2) 1.71, 1.40 and 1.36
(3) 1.71, 1.36 and 1.40
(4) 1.36, 1.40 and 1.71

Sol. **2**
Ionic radii
$N^{3-} > O^{-2} > F^{-}$

5. The color of $KMnO_4$ is due to:
(1) d – d transition
(2) L → M charge transfer transition
(3) $\sigma - \sigma^*$ transition
(4) M → L charge transfer transition

Sol. **2**
L → M charge transfer transition

6. **Assertion :** Nitrogen and Oxygen are the main components in the atmosphere but these do not react to form oxides of nitrogen.
Reason : The reaction between nitrogen and oxygen requires high temperature.
(1) Both assertion and reason are correct, but the reason is not the correct explanation for the assertion
(2) The assertion is incorrect, but the reason is correct
(3) Both the assertion and reason are incorrect
(4) Both assertion and reason are correct, and the reason is the correct explanation for the assertion

Sol. **4**

7. Which of the following compounds is not an antacid?
(1) Cimetidine
(2) Phenelzine
(3) Ranitidine
(4) Aluminium hydroxide

Sol. **2**
Phenelzine is anti-depressant

8. In the context of the Hall-Heroult process for the extraction of A*l*, which of the following statements is false?
(1) Al_2O_3 is mixed with CaF_2 which lowers the melting point of the mixture and brings conductivity
(2) Al^{3+} is reduced at the cathode to form Al
(3) Na_3AlF_6 serves as the electrolyte
(4) CO and CO_2 are produced in this process

Sol. **3**
* $2Al_2O_3 + 3C \longrightarrow 4Al + 3CO_2$
* Na_3AlF_6 or CaF_2 is mixed with purified Al_2O_3 to lower the melting point and brings conductivity.
* Oxygen librated at anode reacts with carbon anode to librate CO and CO_2.

9. Match the catalysts to the correct processes:

	Catalyst		**Process**
(A)	$TiCl_3$	(i)	Wacker process
(B)	$PdCl_2$	(ii)	Ziegler – Natta polymerization
(C)	$CuCl_2$	(iii)	Contact process
(D)	V_2O_5	(iv)	Deacon's process

(1) (A) – (ii), (B) – (i), (C) – (iv), (D) – (iii)
(2) (A) – (ii), (B) – (iii), (C) – (iv), (D) – (i)
(3) (A) – (iii), (B) – (i), (C) – (ii), (D) – (iv)
(4) (A) – (iii), (B) – (ii), (C) – (iv), (D) – (i)

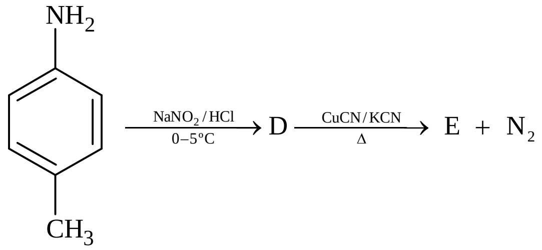
Sol. **1**
$TiCl_4 + (C_2H_5)_3Al \rightarrow$ Ziegler Natta catalyst, used for coordination polymerization.

$$CH_2 = CH_2 \xrightarrow[H_2O + PdCl_2]{\text{Wacker process}} CH_3 - CHO + 2HCl$$

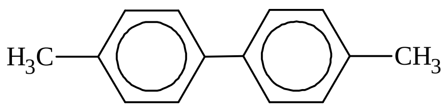
$CuCl_2 \rightarrow$ used as catalyst in Deacon's process of production of Cl_2.

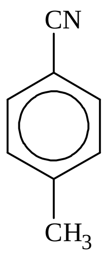
$V_2O_5 \rightarrow$ used as catalyst in Contact Process of manufacturing of H_2SO_4.

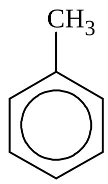
10. In the reaction:

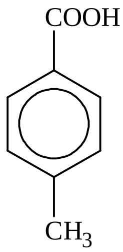


the product E is

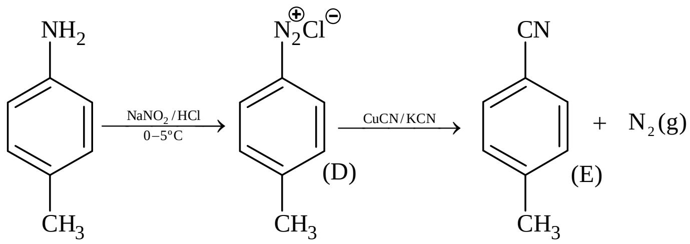
(1) 

(2) 

(3) 

(4) 

Sol. **2**



11. Which polymer is used in the manufacture of paints and lacquers?

(1) Glyptal
(2) Polypropene
(3) Poly vinyl chloride
(4) Bakelite

Sol. **1**

Glyptal $\rightarrow$ Manufacture of paints and lacquers

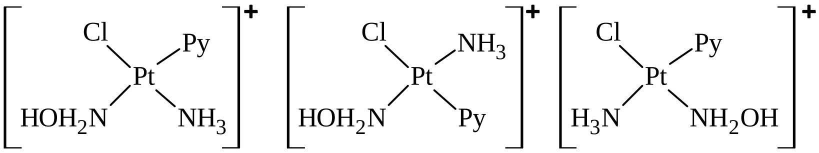
Polypropene $\rightarrow$ Manufacture of ropes, toys

Poly vinyl chloride $\rightarrow$ Manufacture of raincoat, handbags

Bakelite $\rightarrow$ Making combs and electrical switch

12. The number of geometric isomers that can exist for square planar $[Pt(Cl)(py)(NH_3)(NH_2OH)]^+$ is (py = pyridine):
(1) 3 (2) 4
(3) 6 (4) 2

Sol. **1**



13. Higher order (>3) reactions are rare due to:
(1) increase in entropy and activation energy as more molecules are involved
(2) shifting of equilibrium towards reactants due to elastic collisions
(3) loss of active species on collision
(4) low probability of simultaneous collision of all the reacting species

Sol. **4**
Higher order (>3) reactions are less probable due to low probability of simultaneous collision of all the reacting species.

14. Which among the following is the most reactive?
(1) Br_2 (2) I_2
(3) ICl (4) Cl_2

Sol. **3**
Inter halogen compounds are more reactive than corresponding halogen molecules due to polarity of bond.

15. Two Faraday of electricity is passed through a solution of $CuSO_4$. The mass of copper deposited at the cathode is: (at. mass of Cu = 63.5 amu)
(1) 63.5 g (2) 2 g
(3) 127 g (4) 0 g

Sol. **1**
$Cu^{2+} + 2e^- \longrightarrow Cu$
2F of electricity will give 1 mole of Cu.

*16. 3 g of activated charcoal was added to 50 mL of acetic acid solution (0.06N) in a flask. After an hour it was filtered and the strength of the filtrate was found to be 0.042 N. The amount of acetic acid adsorbed (per gram of charcoal) is:
(1) 36 mg (2) 42 mg
(3) 54 mg (4) 18 mg

Sol. **4**
Amount adsorbed
$= (0.060 - 0.042) \times 50 \times 10^{-3} \times 60$
$= 0.018 \times 50 \times 60 \times 10^{-3}$
$= 0.018 \times 3$
$= 0.054$ gm = 54 mg
Amount adsorbed per gram of activated charcoal $= \frac{54}{3} = 18$ mg

17. The synthesis of alkyl fluorides is best accomplished by:
(1) Sandmeyer's reaction (2) Finkelstein reaction
(3) Swarts reaction (4) Free radical fluorination

Sol. **3**

$$2R-Cl + Hg_2F_2 \xrightarrow{\Delta} 2R-F + Hg_2Cl_2$$

*18. The molecular formula of a commercial resin used for exchanging ions in water softening is $C_8H_7SO_3Na$(Mol. Wt. 206). What would be the maximum uptake of Ca^{2+} ions by the resin when expressed in mole per gram resin?

(1) $\frac{1}{206}$ (2) $\frac{2}{309}$

(3) $\frac{1}{412}$ (4) $\frac{1}{103}$

Sol. **3**

$$2C_8H_7SO_3Na \;+\; Ca^{+2} \longrightarrow (C_8H_7SO_3)_2Ca$$

2 mole 1 mole

412 gm 1 mole

Maximum uptake of Ca^{+2} ions by the resin = 1/412 (mole per gm resin)

19. Which of the vitamins given below is water soluble?

(1) Vitamin D (2) Vitamin E

(3) Vitamin K (4) Vitamin C

Sol. **4**

Vitamin C is water soluble

*20. The intermolecular interaction that is dependent on the inverse cube of distance between the molecule is:

(1) ion-dipole interaction (2) London force

(3) hydrogen bond (4) ion-ion interaction

Sol. **1**

For ion-dipole interaction

$F \propto \mu \frac{dE}{dr}$ (where μ is dipole moment of dipole and r is distance between ion and dipole)

$$\propto \mu \frac{d}{dr}\left(\frac{1}{r^2}\right)$$

$$\propto \frac{\mu}{r^3}$$

*21. The following reaction is performed at 298 K.

$$2NO(g) + O_2(g) \rightleftharpoons 2NO_2(g)$$

The standard free energy of formation of NO(g) is 86.6 kJ/mol at 298 K. What is the standard free energy of formation of NO_2(g) at 298 K? ($K_P = 1.6 \times 10^{12}$)

(1) $86600 + R(298)\ln(1.6\times10^{12})$ (2) $86600 - \frac{\ln(1.6\times10^{12})}{R(298)}$

(3) $0.5[2\times86,600 - R(298)\ln(1.6\times10^{12})]$ (4) $R(298)\ln(1.6\times10^{12}) - 86600$

Sol. **3**

$$\Delta G^o_{Rex^n} = 2\Delta G^o_f(NO_2) - 2\Delta G^o_f(NO) - \Delta G^o_f(O_2)$$

$$2\Delta G^o_f(NO_2) = \Delta G^o_{(Rex^n)} + 2\Delta G^o_f(NO) + \Delta G^o_f(O_2)$$

$$\Delta G = \Delta G^o + RT\ln k_p$$

At equilibrium,

$\Delta G = 0, \; Q = k_p$

$\Delta G^o = -RT \ln k_p$

$\Delta G^o(O_2) = 0$

$\Delta G_f^0(NO_2) = 0.5[2\times86{,}600 - R(298)\ln(1.6\times10^{12})]$

22. Which of the following compounds is **not** colored yellow?

(1) $K_3[Co(NO_2)_6]$ (2) $(NH_4)_3[As(Mo_3O_{10})_4]$

(3) $BaCrO_4$ (4) $Zn_2[Fe(CN)_6]$

Sol. **4**

$Zn_2[Fe(CN)_6]$ is bluish white and rest are yellow colored compounds.

*23. In Carius method of estimation of halogens, 250 mg of an organic compound gave 141 mg of AgBr. The percentage of bromine in the compound is: *(at. Mass Ag = 108; Br = 80)*

(1) 36 (2) 48

(3) 60 (4) 24

Sol. **4**

Let the organic compound is R – Br

$R - Br + AgNO_3 \longrightarrow AgBr\downarrow$

So number of moles of AgBr ≡ Number of moles of R – Br ≡ Number of moles of Br

$\frac{141\times10^{-3}}{188}$ = number of moles of Br = 0.75×10^{-3}

Mass of bromine = $0.75 \times 80 \times 10^{-3}$ g = 60 mg

Percentage of bromine = $\frac{60}{250}\times100$ = 24 %

24. Sodium metal crystallizes in a body centred cubic lattice with a unit cell edge of 4.29 Å. The radius of sodium atom is approximately:

(1) 3.22 Å (2) 5.72 Å

(3) 0.93 Å (4) 1.86 Å

Sol. **4**

For body center unit cell.

$4r = a\sqrt{3}$

So radius of Na = $\frac{\sqrt{3}\times4.29}{4} = 1.857 \approx 1.86$ Å

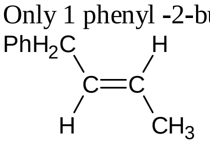
*25. Which of the following compounds will exhibit geometrical isomerism?

(1) 3-Phenyl-1-butene (2) 2-Phenyl-1-butene

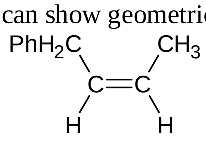
(3) 1,1-Diphenyl-1-propane (4) 1-Phenyl-2-butene

Sol. **4**

Only 1 phenyl -2-butene can show geometrical isomerism.



and



trans cis

26. The vapour pressure of acetone at 20°C is 185 torr. When 1.2 g of a non-volatile substance was dissolved in 100 g of acetone at 20°C, its vapour pressure was 183 torr. The molar mass (g mol^{-1}) of the substance is:
(1) 64 (2) 128
(3) 488 (4) 32

Sol. **1**
Vapour pressure of pure acetone (p^0) = 185 torr
Vapour pressure of solution (p) = 183 torr
Then from Raoult's law,

$$\frac{p^0 - p}{p} = \frac{n_{solute}}{n_{solvent}}$$

$$\frac{185 - 183}{183} = \frac{1.2 / MW}{100 / 58}$$

$$\frac{2}{183} = \frac{1.2 \times 58}{100 \times MW}$$

$$MW = \frac{1.2 \times 58 \times 183}{200} = 63.68 \approx 64$$

*27. From the following statements regarding H_2O_2, choose **the incorrect** statement:
(1) It decomposes on exposure to light
(2) It has to be stored in plastic or wax lined glass bottles in dark
(3) It has to be kept away from dust
(4) It can act only as an oxidizing agent

Sol. **4**
H_2O_2 can act both as oxidizing agent as well as reducing agent and H_2O_2 decomposes on exposures to light and dust; so as to kept in plastic or wax lined glass bottles in dark.

*28. Which one of the following alkaline earth metal sulphates has its hydration enthalpy greater than its lattice enthalpy?
(1) $BeSO_4$ (2) $BaSO_4$
(3) $SrSO_4$ (4) $CaSO_4$

Sol. **1**
Because of smallest size of Be^{2+}, its hydration energy is maximum and is greater than the lattice energy of $BeSO_4$.

*29. The standard Gibbs energy change at 300 K for the reaction $2A \rightleftharpoons B + C$ is 2494.2 J. At a given time, the composition of the reaction mixture is $[A] = \frac{1}{2}, [B] = 2$ and $[C] = \frac{1}{2}$. The reaction proceeds in the:
[R = 8.314 J/K/mol, e = 2.718]
(1) reverse direction because $Q > K_c$
(2) forward direction because $Q < K_c$
(3) reverse direction because $Q < K_c$
(4) forward direction because $Q > K_c$

Sol. **1**

$$\Delta G^0 = -RT \ln K$$

$$2494.2 = -8.314 \times 300 \ln K$$

$$\ln K = -1$$

$$\log_{2.718} K = -1$$

$K = (2.718)^{-1} = \frac{1}{2.718} \quad = 0.3679$

$Q_c = \frac{[B][C]}{[A]^2} = \frac{2\times\frac{1}{2}}{\left(\frac{1}{2}\right)^2} = 4$

$Q_c > K_c \Rightarrow$ Reverse direction.

30. Which one has the highest boiling point?

(1) Ne (2) Kr
(3) Xe (4) He

Sol. **3**

The boiling point of Noble gases in increasing order:

He < Ne < Ar < Kr < Xe < Rn

Boiling point (K) : 4.2< 27.1< 87.2< 119.7< 165< 211

PART B – MATHEMATICS

*31. The sum of coefficients of integral powers of x in the binomial expansion of $\left(1-2\sqrt{x}\right)^{50}$ is :

(1) $\frac{1}{2}\left(3^{50}\right)$ (2) $\frac{1}{2}\left(3^{50}-1\right)$
(3) $\frac{1}{2}\left(2^{50}+1\right)$ (4) $\frac{1}{2}\left(3^{50}+1\right)$

Sol. **4**

$t_{r+1} = {}^{50}C_r \cdot (1)^{50-r} \cdot \left(-2x^{1/2}\right)^r$

$= {}^{50}C_r \cdot 2^r \cdot x^{r/2} (-1)^r \Rightarrow r =$ even integer.

$\Rightarrow$ Sum of coefficient $= \sum_{r=0}^{25} {}^{50}C_{2r} \cdot 2^{2r} = \frac{1}{2}\left((1+2)^{50} + (1-2)^{50}\right) = \frac{1}{2}\left(3^{50}+1\right)$

32. Let f(x) be a polynomial of degree four having extreme values at x = 1 and x = 2. If $\lim_{x\to 0}\left[1+\frac{f(x)}{x^2}\right] = 3$, then f(2) is equal to :

(1) – 4 (2) 0
(3) 4 (4) – 8

Sol. **2**

$\lim_{x\to 0}\left(\frac{x^2+f(x)}{x^2}\right) = 3$, since limit exits hence $x^2 + f(x) = ax^4 + bx^3 + 3x^2$

$\Rightarrow \quad f(x) = ax^4 + bx^3 + 2x^2$

$\Rightarrow \quad f'(x) = 4ax^3 + 3bx^2 + 4x$

also $f'(x) = 0$ at x = 1, 2

$\Rightarrow \quad a = \frac{1}{2}, b = -2$

$\Rightarrow \quad f(x) = \frac{x^4}{2} - 2x^3 + 2x^2$

$\Rightarrow \quad f(x) = 8 - 16 + 8 = 0.$

*33. The mean of the data set comprising of 16 observations is 16. If one of the observation valued 16 is deleted and three new observations valued 3, 4 and 5 are added to the data, then the mean of the resultant data, is :

(1) 16.0 (2) 15.8
(3) 14.0 (4) 16.8

Sol. **3**

New sum $\sum y_i = (16\times16-16)+(3+4+5)$ = 252

Number of observation = 18

$\Rightarrow$ New mean $\Rightarrow$ $\overline{y} = \frac{252}{18} = 14$.

*34. The sum of first 9 terms of the series $\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+.......$ is :

(1) 96 (2) 142
(3) 192 (4) 71

Sol. **1**

$t_r = \frac{\sum r^3}{\sum(2r-1)} = \frac{r^2(r+1)^2}{4r^2} = \frac{1}{4}(r+1)^2$

$S_9 = \frac{1}{4}\sum_{r=1}^{9}(r+1)^2$, let t = r + 1

$= \frac{1}{4}\left(\sum_{t=1}^{10}t^2 - 1\right)$ = 96.

*35. Let O be the vertex and Q be any point on the parabola, $x^2 = 8y$. If the point P divides the line segment OQ internally in the ratio 1 : 3, then the locus of P is :

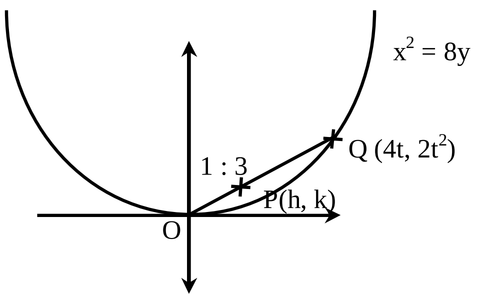
(1) $y^2 = x$ (2) $y^2 = 2x$
(3) $x^2 = 2y$ (4) $x^2 = y$

Sol. **3**

$h = \frac{4t}{4} = t$

$k = \frac{2t^2}{4} = \frac{t^2}{2}$

$\Rightarrow$ $x^2 = 2y$



*36. Let α and β be the roots of equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$, for $n \geq 1$, then the value of $\frac{a_{10}-2a_8}{2a_9}$ is equal to :

(1) – 6 (2) 3
(3) – 3 (4) 6

Sol. **2**

$x^2 = 6x + 2 \Rightarrow \alpha^2 = 6\alpha + 2$

$\Rightarrow \alpha^{10} = 6\alpha^9 + 2\alpha^8$ … (1)

and $\beta^{10} = 6\beta^9 + 2\beta^8$ … (2)

$\Rightarrow$ Subtract (2) from (1)

$a_{10} = 6a_9 + 2a_8$

$\Rightarrow \dfrac{a_{10} - 2a_8}{2a_9} = 3.$

37. If 12 identical balls are to be placed in 3 identical boxes, then the probability that one of the boxes contains exactly 3 balls is :

(1) $55\left(\dfrac{2}{3}\right)^{10}$ (2) $220\left(\dfrac{1}{3}\right)^{12}$

(3) $22\left(\dfrac{1}{3}\right)^{11}$ (4) $\dfrac{55}{3}\left(\dfrac{2}{3}\right)^{11}$

Sol. Correct option is not available

Required probability $= \left(\dfrac{{}^3C_1\,{}^{12}C_3\,2^9 - {}^3C_2\,{}^{12}C_3\,{}^9C_3}{3^{12}}\right)$

*38. A complex number z is said to be unimodular if |z| = 1. Suppose z_1 and z_2 are complex numbers such that $\dfrac{z_1 - 2z_2}{2 - z_1\bar{z}_2}$ is unimodular and z_2 is not unimodular. Then the point z_1 lies on a :

(1) straight line parallel to y-axis. (2) circle of radius 2.

(3) circle of radius $\sqrt{2}$. (4) straight line parallel to x-axis.

Sol. **2**

$\left|\dfrac{z_1 - 2z_2}{z - z_1\bar{z}_2}\right| = 1$

$(z_1 - 2z_2)(\bar{z}_1 - 2\bar{z}_2)$

$= (2 - z_1\bar{z}_2)(2 - \bar{z}_1 z_2)$

$\Rightarrow |z_1|^2 - 2z_2\bar{z}_1 - 2\bar{z}_2 z_1 + 4|z_2|^2$

$= 4 - 2z_1\bar{z}_2 - 2\bar{z}_1 z_2 + |z_1|^2\,|z_2|^2$

$\Rightarrow |z_1|^2 + 4|z_2|^2 - 4 - |z_1|^2\,|z_2|^2 = 0$

$|z_1|^2\left(1 - |z_2|^2\right) - 4\left(1 - |z_2|^2\right) = 0$

$\Rightarrow |z_1| = 2$ (as $|z_2| \neq 1$)

39. The integral $\displaystyle\int \frac{dx}{x^2\left(x^4+1\right)^{3/4}}$ equals :

(1) $(x^4 + 1)^{1/4} + c$ (2) $-(x^4 + 1)^{1/4} + c$

(3) $-\left(\dfrac{x^4+1}{x^4}\right)^{1/4} + c$ (4) $\left(\dfrac{x^4+1}{x^4}\right)^{1/4} + c$

Sol. **3**

$\displaystyle\int \frac{dx}{x^5\left(1 + \frac{1}{x^4}\right)^{3/4}}$

$1+\frac{1}{x^4}=t$

$\frac{-4}{x^5}dx=dt$

$\Rightarrow -\frac{1}{4}\int\frac{1}{t^{3/4}}dt$

$= -\frac{1}{4}\times 4t^{1/4}+c = -\left(1+\frac{1}{x^4}\right)^{1/4}+c$.

*40. The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices (0, 0), (0, 41) and (41, 0), is :

(1) 861 (2) 820
(3) 780 (4) 901

Sol. **3**

$x + y < 41$, $x > 0$, $y > 0$ is bounded region.
Number of positive integral solutions of the equation $x + y + k = 41$ will be number of integral co-ordinates in the bounded region.
$\Rightarrow {}^{41-1}C_{3-1} = {}^{40}C_2 = 780$.

41. The distance of the point (1, 0, 2) from the point of intersection of the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ and the plane $x - y + z = 16$, is :

(1) 8 (2) $3\sqrt{21}$
(3) 13 (4) $2\sqrt{14}$

Sol. **3**

Let the point of intersection be $(2 + 3\lambda, 4\lambda - 1, 12\lambda + 2)$
$(2 + 3\lambda) - (4\lambda - 1) + 12\lambda + 2 = 16$
$11\lambda = 11$
$\lambda = 1$
$\Rightarrow$ point of intersection is (5, 3, 14)
$\Rightarrow$ distance = $\sqrt{(5-1)^2+9+12^2}$
$= \sqrt{16+9+144} = 13$.

42. The equation of the plane containing the line $2x - 5y + z = 3$; $x + y + 4z = 5$, and parallel to the plane, $x + 3y + 6z = 1$, is :

(1) $x + 3y + 6z = -7$ (2) $x + 3y + 6z = 7$
(3) $2x + 6y + 12z = -13$ (4) $2x + 6y + 12z = 13$

Sol. **2**

Let equation of plane is $(2x - 5y + z - 3) + \lambda(x + y + 4z - 5) = 0$
As plane is parallel to $x + 3y + 6z - 1 = 0$

$\frac{2+\lambda}{1}=\frac{\lambda-5}{3}=\frac{1+4\lambda}{6}$

$\Rightarrow 6 + 3\lambda = \lambda - 5$
$11 = -2\lambda$
$\lambda = -\frac{11}{2}$
Also, $6\lambda - 30 = 3 + 12\lambda$

$-6\lambda = 33$

$\lambda = -\frac{11}{2}$.

so the equation of required plane is

$(4x - 10y + 2z - 6) - 11(x + y + 4z - 5) = 0$

$\Rightarrow -7x - 21y - 42z + 49 = 0$

$\Rightarrow x + 3y + 6z - 7 = 0.$

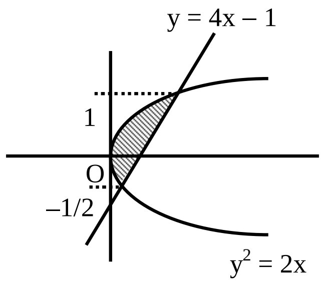
43. The area (in sq. units) of the region described by $\{(x, y) : y^2 \le 2x \text{ and } y \ge 4x - 1\}$ is :

(1) $\frac{5}{64}$ (2) $\frac{15}{64}$

(3) $\frac{9}{32}$ (4) $\frac{7}{32}$

Sol. **3**

The required region



$$= \int_{-\frac{1}{2}}^{1} \left(\frac{y+1}{4} - \frac{y^2}{2} \right) dy$$

$$= \frac{1}{4}\left(\frac{y^2}{2} + y \right)_{-\frac{1}{2}}^{1} - \frac{1}{2}\left(\frac{y^3}{3} \right)_{-\frac{1}{2}}^{1}$$

$$= \frac{1}{4}\left(\frac{1}{2} + 1 - \left(\frac{1}{8} - \frac{1}{2} \right) \right) - \frac{1}{2}\frac{1}{3}\left(1 + \frac{1}{8} \right)$$

$$= \frac{1}{4} \times \frac{15}{8} - \frac{3}{16} = \frac{9}{32}.$$

*44. If m is the A.M. of two distinct real numbers ℓ and n (ℓ, n > 1) and G_1, G_2 and G_3 are three geometric means between ℓ and n, then $G_1^4 + 2G_2^4 + G_3^4$ equals :

(1) $4\ell m^2 n$ (2) $4\ell m n^2$

(3) $4\ell^2 m^2 n^2$ (4) $4\ell^2 mn$

Sol. **1**

Given

$l + n = 2m$ … (i)

l, G_1, G_2, G_3, n are in G.P.

$\Rightarrow$ $G_1 = lr$ (let r be the common ratio)

$G_2 = lr^2$

$G_3 = lr^3$

$n = lr^4$

$$r = \left(\frac{n}{l} \right)^{1/4}$$

$\Rightarrow$ $G_1^4 + 2G_2^4 + G_3^4 = (lr)^4 + 2(lr^2)^4 + (lr^3)^4$

$= l^4 \times r^4 [1 + 2r^4 + r^8]$

$= l^4 \times r^4 [r^4 + 1]^2$

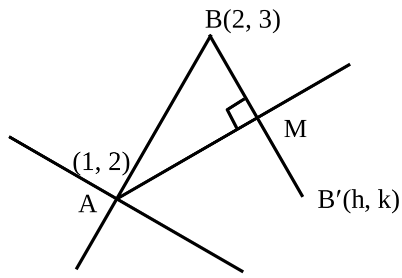
$$= l^4 \times \frac{n}{l} \left[\frac{n+l}{l} \right]^2$$

$= ln \times 4m^2$

$= 4lnm^2.$

*45. Locus of the image of the point (2, 3) in the line (2x – 3y + 4) + k(x – 2y + 3) = 0, k ∈ R, is a :
(1) straight line parallel to y-axis. (2) circle of radius $\sqrt{2}$.
(3) circle of radius $\sqrt{3}$. (4) straight line parallel to x-axis.

Sol. **2**
Let M is mid point of BB′ and AM is ⊥ bisector of BB′ (where A is the point of intersection given lines)



$(x-2)(x-1) + (y-2)(y-3) = 0$

$\Rightarrow \left(\frac{h+2}{2}-2\right)\left(\frac{h+2}{2}-1\right)+\left(\frac{k+3}{2}-2\right)\left(\frac{k+3}{2}-3\right) = 0$

$\Rightarrow (h-2)(h) + (k-1)(k-3) = 0$

$\Rightarrow x^2 - 2x + y^2 - 4y + 3 = 0$

$\Rightarrow (x-1)^2 + (y-2)^2 = 2.$

*46. The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latera recta to the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$, is :
(1) 18 (2) $\frac{27}{2}$
(3) 27 (4) $\frac{27}{4}$

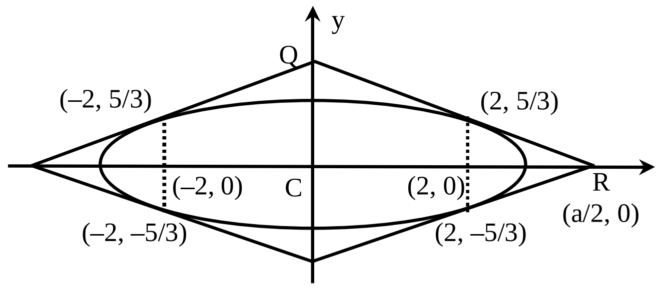
Sol. **3**

$A = 3, b = \sqrt{5}$

$e = \sqrt{1-\frac{5}{9}} = \frac{2}{3}$

foci = (±2, 0)

tangent at P $\Rightarrow \frac{2x}{9}+\frac{5y}{3.5}=1$



$\frac{2x}{9}+\frac{y}{3}=1$

2x + 3y = 9

Area of quadrilateral

= 4 × (area of triangle QCR)

$=\left(\frac{1}{2}\times\frac{9}{2}\times 3\right)\times 4 = 27$

*47. The number of integers greater than 6,000 that can be formed, using the digits 3, 5, 6, 7 and 8, without repetition, is :
(1) 192 (2) 120
(3) 72 (4) 216

Sol. **1**
Four digit numbers will start from 6, 7, 8
3 × 4 × 3 × 2 = 72
Five digit numbers = 5! = 120
Total number of integers = 192.

*48. Let A and B be two sets containing four and two elements respectively. Then the number of subsets of the set A × B, each having at least three elements is :
(1) 256 (2) 275
(3) 510 (4) 219

Sol. **4**

n(A) = 4, n(B) = 2

n(A × B) = 8

number of subsets having atleast 3 elements

$= 2^8 - \left(1 + {}^8C_1 + {}^8C_2\right) = 219$

*49. Let $\tan^{-1} y = \tan^{-1} x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where $|x| < \frac{1}{\sqrt{3}}$. Then a value of y is :

(1) $\frac{3x + x^3}{1 - 3x^2}$

(2) $\frac{3x - x^3}{1 + 3x^2}$

(3) $\frac{3x + x^3}{1 + 3x^2}$

(4) $\frac{3x - x^3}{1 - 3x^2}$

Sol. **4**

$\tan^{-1}y = \tan^{-1}x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$= \tan^{-1}\left(\frac{x + \frac{2x}{1-x^2}}{1 - x\left(\frac{2x}{1-x^2}\right)}\right)$

$= \tan^{-1}\left(\frac{x - x^3 + 2x}{1 - x^2 - 2x^2}\right)$

$\tan^{-1}y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$

$y = \frac{3x - x^3}{1 - 3x^2}$.

50. The integral $\int_2^4 \frac{\log x^2}{\log x^2 + \log\left(36 - 12x + x^2\right)} dx$ is equal to :

(1) 4

(2) 1

(3) 6

(4) 2

Sol. **2**

Apply the property

$\int_a^b f(x)dx = \int_a^b f(a + b - x)dx$

And then add

$2I = \int_2^4 1dx$

2I = 2

I = 1.

*51. The negation of $\sim s \vee (\sim r \wedge s)$ is equivalent to :

(1) $s \wedge (r \wedge \sim s)$

(2) $s \vee (r \vee \sim s)$

(3) $s \wedge r$

(4) $s \wedge \sim r$

Sol. **3**

$\sim (\sim s \vee (\sim r \wedge s))$
$\equiv s \wedge (\sim (\sim r \wedge s))$
$\equiv s \wedge (r \vee \sim s) \equiv (s \wedge r) \vee (s \wedge \sim s)$
$\equiv (s \wedge r) \vee F$
$\equiv s \wedge r.$

*52. If the angles of elevation of the top of a tower from three collinear points A, B and C, on a line leading to the foot of the tower, are 30°, 45° and 60° respectively, then the ratio, AB : BC, is :

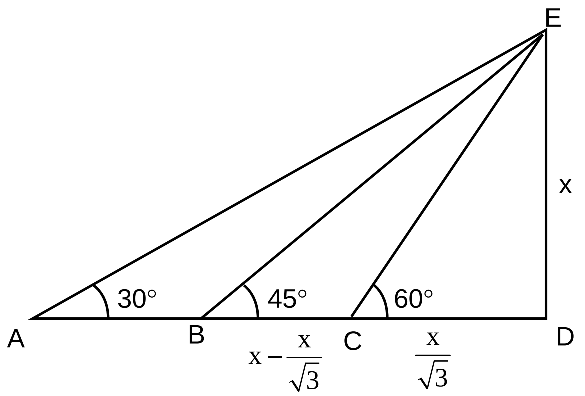
(1) $\sqrt{3} : \sqrt{2}$ (2) $1 : \sqrt{3}$
(3) 2 : 3 (4) $\sqrt{3} : 1$

Sol. **4**

AB = $\sqrt{3}x - x$

BC = $x - \frac{x}{\sqrt{3}}$

$\frac{AB}{BC} = \frac{\sqrt{3}x - x}{x - \frac{x}{\sqrt{3}}} = \frac{\sqrt{3}}{1}$.



53. $\lim_{x \to 0} \frac{(1 - \cos 2x)(3 + \cos x)}{x \tan 4x}$ is equal to :

(1) 3 (2) 2
(3) $\frac{1}{2}$ (4) 4

Sol. **2**

$\lim_{x \to 0} \frac{2\sin^2 x \times (3 + \cos x)}{x \times \left(\frac{\tan 4x}{4x}\right) \times 4x}$

$= \frac{2 \times 4}{4} = 2$.

54. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of them are collinear and $(\vec{a} \times \vec{b}) \times \vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$.

If θ is the angle between vectors $\vec{b}$ and $\vec{c}$, then a value of sinθ is :

(1) $\frac{-\sqrt{2}}{3}$ (2) $\frac{2}{3}$
(3) $\frac{-2\sqrt{3}}{3}$ (4) $\frac{2\sqrt{2}}{3}$

Sol. **4**

$(\vec{a} \cdot \vec{c})\vec{b} - (\vec{b} \cdot \vec{c})\vec{a} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

$\Rightarrow -\ \vec{b}\cdot\vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|$

$\Rightarrow\ -|\vec{b}||\vec{c}|\cos\theta = \frac{1}{3}|\vec{b}||\vec{c}|$

$\Rightarrow \cos\theta = -\frac{1}{3}$

$\Rightarrow \sin\theta = \frac{2\sqrt{2}}{3}$.

55. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying the equation $AA^T = 9I$, where I is 3×3 identity matrix, then ordered pair (a, b) is equal to :

(1) (–2, 1) (2) (2, 1)
(3) (–2, –1) (4) (2, –1)

Sol. **3**

$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$, $A^T = \begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix}$

$AA^T = \left[b_{ij}\right]_{3\times3}$

$b_{13} = 0 \Rightarrow 0 = a + 4 + 2b$

$b_{23} = 0 \Rightarrow 0 = 2a + 2 - 2b$

$\Rightarrow 3a + 6 = 0 \Rightarrow a = -2,\ b = -1$.

56. If the function. $g(x) = \begin{cases} k\sqrt{x+1}\ , & 0 \le x \le 3 \\ mx+2\ , & 3 < x \le 5 \end{cases}$ is differentiable, then the value of k + m is :

(1) $\frac{16}{5}$ (2) $\frac{10}{3}$
(3) 4 (4) 2

Sol. **4**

for f(x) to be continuous

$2k = 3m + 2$

$2k - 3m = 2$ … (i)

for f(x) to be differentiable

$\frac{k}{4} = m$

$k = 4m$.

from (i), $8m - 3m = 2$

$5m = 2$

$m = \frac{2}{5}$

$k = 4\times\frac{2}{5} = \frac{8}{5}$

$k + m = \frac{2}{5} + \frac{8}{5} = \frac{10}{5} = 2$.

57. The set of all values of λ for which the system of linear equations :

$$2x_1 - 2x_2 + x_3 = \lambda x_1$$
$$2x_1 - 3x_2 + 2x_3 = \lambda x_2$$
$$-x_1 + 2x_2 = \lambda x_3$$

has a non-trivial solution,

(1) is a singleton.
(2) contains two elements.
(3) contains more than two elements.
(4) is an empty set.

Sol. **2**

$(2 - \lambda)\, x_1 - 2x_2 + x_3 = 0$
$2x_1 - (3 + \lambda)\, x_2 + 2x_3 = 0$
$- x_1 + 2x_2 - \lambda x_3 = 0$

$$\Rightarrow \begin{vmatrix} 2-\lambda & -2 & 1 \\ 2 & -3-\lambda & 2 \\ -1 & 2 & -\lambda \end{vmatrix} = 0$$

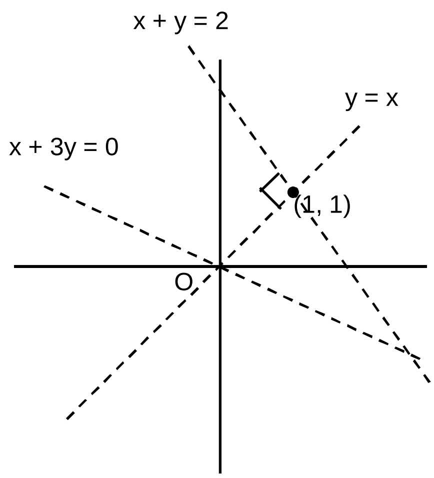
$(2 - \lambda)\,(3\lambda + \lambda^2 - 4) + 2\,(- 2\lambda + 2) + 1\,(4 - 3 - \lambda) = 0$
$(2 - \lambda)\,(\lambda^2 + 3\lambda - 4) + 4\,(1 - \lambda) + (1 - \lambda) = 0$
$(2 - \lambda)\,((\lambda + 4)\,(\lambda - 1)) + 5\,(1 - \lambda) = 0$
$(1 - \lambda)\,((\lambda + 4)\,(\lambda - 2) + 5) = 0 \Rightarrow \lambda = 1, 1, - 3.$

*58. The normal to the curve, $x^2 + 2xy - 3y^2 = 0$, at (1, 1) :

(1) meets the curve again in the second quadrant.
(2) meets the curve again in the third quadrant.
(3) meets the curve again in the fourth quadrant.
(4) does not meet the curve again.

Sol. **3**

$X^2 + 3xy - xy - 3y^2 = 0$
$x\,(x + 3y) - y\,(x + 3y) = 0$
$(x + 3y)\,(x - y) = 0$
Equation of normal is $(y - 1) = - 1\,(x - 1)$
$\Rightarrow x + y = 2$
It intersects $x + 3y = 0$ at $(3, - 1)$
And hence meets the curve again in the 4th quadrant.



*59. The number of common tangents to the circles $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$, is :

(1) 2
(2) 3
(3) 4
(4) 1

Sol. **2**

$C_1\,(2, 3)$; $\quad r_1 = \sqrt{4+9+12} = 5$
and $C_2\,(- 3, - 9)$; $\quad r_2 = \sqrt{9+81-26} = 8$
$C_1C_2 = \sqrt{25+144} = 13$
$C_1C_2 = = r_1 + r_2$ touching externally.

$\Rightarrow$ 3 common tangents.

60. Let y(x) be the solution of the differential equation $(x \log x)\frac{dy}{dx} + y = 2x \log x, (x \geq 1)$. Then y(e) is equal to :

(1) 0 (2) 2
(3) 2e (4) e

Sol. **2**

$\frac{dy}{dx} + \frac{y}{x \ln x} = \frac{2x \ln x}{x \ln x}$

I.F. = $e^{\int \frac{1}{x \ln x} dx} = e^{\ln(\ln x)} = \ln x$

y ln x = $\int 2 \ln x \, dx$

y ln x = 2 (x ln x – x) + c

For x = 1, c = 2

y lnx = 2 (x lnx – x + 1)

put x = e $\Rightarrow$ y(e) = 2.

PART C – PHYSICS

61. As an electron makes a transition from an excited state to the ground state of a hydrogen – like atom/ion:

(1) kinetic energy, potential energy and total energy decrease
(2) kinetic energy decreases, potential energy increases but total energy remains same
(3) kinetic energy and total energy decrease but potential energy increases
(4) its kinetic energy increases but potential energy and total energy decrease

Sol. **4**

As electron goes to ground state, total energy decreases.

TE = –KE

PE = 2TE

So, kinetic energy increases but potential energy and total energy decreases.

*62. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using wrist watch of 1 s resolution. The accuracy in the determination of g is:

(1) 3% (2) 1%
(3) 5% (4) 2%

Sol. **1**

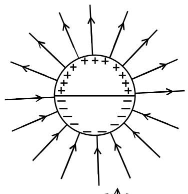
$g = \frac{4\pi^2 L}{T^2}$

$\frac{\Delta g}{g} = \frac{\Delta L}{L} + 2\left(\frac{\Delta T}{T}\right)$

$\frac{\Delta L}{L} = \frac{0.1}{20}, \frac{\Delta T}{T} = \frac{0.01}{0.9}$

$100\left(\frac{\Delta g}{g}\right) = 100\left(\frac{\Delta L}{L}\right) + 2 \times 100 \times \left(\frac{\Delta T}{T}\right) \simeq 3\%$

63. A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge −σ in the lower half. The electric field lines around the cylinder will look like figure given in: (figures are schematic and not drawn to scale)

(1)


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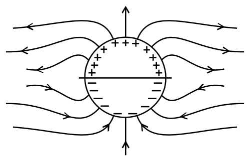

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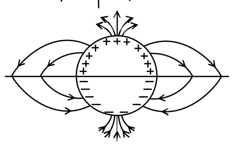

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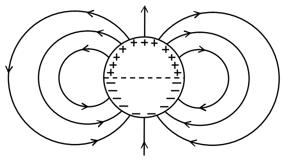


Sol. **4**

It originates from +Ve charge and terminates at −Ve charge. It can not form close loop.

64. A signal of 5 kHz frequency is amplitude modulated on a carrier wave of frequency 2 MHz. The frequencies of the resultant signal is/are:

(1) 2005 kHz, and 1995 kHz
(2) 2005 kHz, 2000 kHz and 1995 kHz
(3) 2000 kHz and 1995 kHz
(4) 2 MHz only

Sol. **2**

$f_R = f_C + f_m$ = 2000 kHz + 5kHz = 2005 kHz

$f_R = f_C - f_m$ = 2000 kHz − 5kHz = 1995 kHz

So, frequency content of resultant wave will have frequencies 1995 kHz, 2000 kHz and 2005 kHz

*65. Consider a spherical shell of radius R at temperature T. The black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume $u = \frac{U}{V} \propto T^4$ and pressure $p = \frac{1}{3}\left(\frac{U}{V}\right)$. If the shell now undergoes an adiabatic expansion the relation between T and R is:

(1) $T \propto e^{-3R}$
(2) $T \propto \frac{1}{R}$
(3) $T \propto \frac{1}{R^3}$
(4) $T \propto e^{-R}$

Sol. **2**

$dQ = dU + dW$

$dU = -pdV$

$$\frac{dU}{dV} = -p = -\frac{1}{3}\frac{U}{V}$$

$$\frac{dU}{U} = -\frac{1}{3}\frac{dV}{V}$$

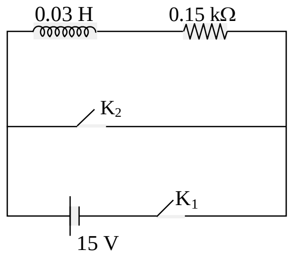
$$\ell nU = -\frac{1}{3}\ell nV + \ell nC$$

$$U \cdot V^{1/3} = C$$

$$VT^4 \cdot V^{1/3} = C'$$

$$T \propto \frac{1}{R}$$

66. An inductor (L = 0.03 H) and a resistor (R = 0.15 kΩ) are connected in series to a battery of 15V EMF in a circuit shown. The key K_1 has been kept closed for a long time. Then at t = 0, K_1 is opened and key K_2 is closed simultaneously. At t = 1 ms, the current in the circuit will be: ($e^5 \cong 150$)



(1) 67 mA
(2) 6.7 mA
(3) 0.67 mA
(4) 100 mA

Sol. **3**

When K_1 is closed and K_2 is open,

$$I_0 = \frac{E}{R}$$

when K_1 is open and K_2 is closed, current as a function of time 't' in L.R. circuit.

$$I = I_0 e^{-R\frac{t}{L}}$$

$$= \frac{1}{10}e^{-5} = \frac{1}{1500} = 0.67 \text{ mA}$$

*67. A pendulum made of a uniform wire of cross sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to T_M. If the Young's modulus of the material of the wire is Y then $\frac{1}{Y}$ is equal to: *(g = gravitational acceleration)*

(1) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{Mg}{A}$
(2) $\left[1 - \left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$
(3) $\left[1 - \left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$
(4) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{A}{Mg}$

Sol. **4**

Time period, $T = 2\pi\sqrt{\frac{\ell}{g}}$

When additional mass M is added to its bob

$$T_M = 2\pi\sqrt{\frac{\ell + \Delta\ell}{g}}$$

$$\Delta\ell = \frac{Mg\ell}{AY}$$

$$\Rightarrow T_M = 2\pi\sqrt{\frac{\ell + \frac{Mg\ell}{AY}}{g}}$$

$$\left(\frac{T_M}{T}\right)^2 = 1 + \frac{Mg}{AY}$$

$$\frac{1}{Y} = \frac{A}{Mg}\left[\left(\frac{T_M}{T}\right)^2 - 1\right]$$

68. A red LED emits light at 0.1 watt uniformly around it. The amplitude of the electric field of the light at a distance of 1 m from the diode is:

(1) 2.45 V/m (2) 5.48 V/m
(3) 7.75 V/m (4) 1.73 V/m

Sol. **1**

Intensity, $I = \frac{1}{2}\varepsilon_0 E_0^2 C$, where E_0 is amplitude of the electric field of the light.

$$\frac{P}{4\pi r^2} = \frac{1}{2}\varepsilon_0 E_0^2 C$$

$$E_0 = \sqrt{\frac{2P}{4\pi r^2 C\varepsilon_0}} = 2.45 \text{ V/m}$$

69. Two coaxial solenoids of different radii carry current I in the same direction. Let $\vec{F}_1$ be the magnetic force on the inner solenoid due to the outer one and $\vec{F}_2$ be the magnetic force on the outer solenoid due to the inner one. Then:

(1) $\vec{F}_1$ is radially inwards and $\vec{F}_2$ is radially outwards

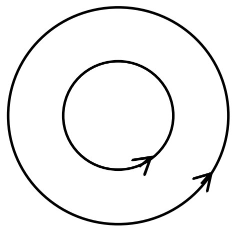
(2) $\vec{F}_1$ is radially inwards and $\vec{F}_2 = 0$

(3) $\vec{F}_1$ is radially outwards and $\vec{F}_2 = 0$

(4) $\vec{F}_1 = \vec{F}_2 = 0$

Sol. **4**

Both solenoid are in equilibrium so, net Force on both solenoids due to other is zero.
So, $F_1 = F_2 = 0$



*70. Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion the average time of collision between molecules increases as V^q, where V is the volume of the gas. The value of q is :

$\left(\gamma = \frac{C_p}{C_v}\right)$

(1) $\frac{3\gamma - 5}{6}$ (2) $\frac{\gamma + 1}{2}$

(3) $\frac{\gamma - 1}{2}$ (2) $\frac{3\gamma + 5}{6}$

Sol. **2**

Average time between collision = $\frac{\text{Mean free Path}}{V_{rms}}$

$t = \frac{\frac{1}{\pi d^2 N/V}}{\sqrt{\frac{3RT}{M}}}$; $t = \frac{CV}{\sqrt{T}}$ (where $C = \frac{\sqrt{M}}{\pi d^2 N\sqrt{3R}} = \text{constant}$)

$\Rightarrow \quad T \propto \frac{V^2}{t^2}$

For adiabatic

$TV^{\gamma-1} = k$

$\frac{V^2}{t^2} V^{\gamma-1} = k$

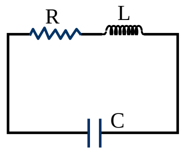
$\frac{V^{\gamma+1}}{t^2} = k$

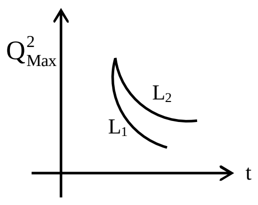
$t \propto V^{\frac{\gamma+1}{2}}$

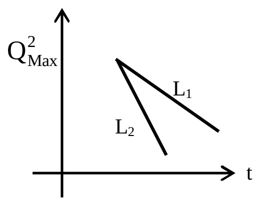
so, $q = \frac{\gamma+1}{2}$

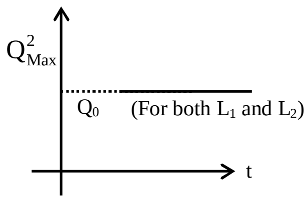
71. An LCR circuit is equivalent to a damped pendulum. In an LCR circuit the capacitor is charged to Q_0 and then connected to the L and R as shown.

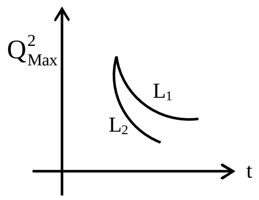
If a student plots graphs of the square of maximum charge (Q^2_{Max}) on the capacitor with time (t) for two different values L_1 and L_2 ($L_1 > L_2$) of L then which of the following represents this graph correctly? (Plots are schematic and not drawn to scale)



(1)


(2)


(3)


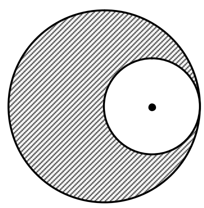
(4)


Sol. **4**

As $L_1 > L_2$, therefore $\frac{1}{2}L_1 i^2 > \frac{1}{2}L_2 i^2$,

$\therefore$ Rate of energy dissipated through R from L_1 will be slower as compared to L_2.

*72. From a solid sphere of mass M and radius R, a spherical portion of radius R/2 is removed, as shown in the figure. Taking gravitational potential V = 0 at r = ∞, the potential at the centre of the cavity thus formed is (G = gravitational constant)



(1) $-\frac{GM}{R}$ (2) $-\frac{2GM}{3R}$

(3) $-\frac{2GM}{R}$ (4) $\frac{-GM}{2R}$

Sol. **1**

$V_{required} = V_M - V_{M/8}$

$= -\frac{GM}{2R^3}\left[3R^2 - \frac{R^2}{4}\right] + \frac{GM/8}{2(R/2)^3}\left[3(R/2)^2\right]$

$= -\frac{11GM}{8R} + \frac{3GM}{8R} = -\frac{GM}{R}$.

*73. A train is moving on a straight track with speed 20 ms^{-1}. It is blowing its whistle at the frequency of 1000 Hz. The percentage change in the frequency heard by a person standing near the track as the train passes him is (speed of sound = 320 ms^{-1}) close to :

(1) 12 % (2) 18 %
(3) 24 % (4) 6 %

Sol. **1**

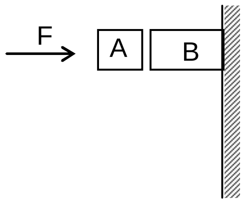
f_1 (train approaches) = $1000\left(\frac{320}{320-20}\right) = 1000\left(\frac{320}{300}\right)$Hz.

f_2(train recedes) = $1000\left(\frac{320}{320+20}\right) = 1000\left(\frac{320}{340}\right)$Hz.

$\Delta f = \left(\frac{f_1 - f_2}{f_1}\right) \times 100\% = \left(1 - \frac{300}{340}\right) \times 100\% = \frac{40}{340} \times 100\%$

= 11.7 % ≈ 12 %.

*74. Given in the figure are two blocks A and B of weight 20 N and 100 N respectively. These are being pressed against a wall by a force F as shown. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force applied by the wall on block B is



(1) 80 N (2) 120 N
(3) 150 N (4) 100 N

Sol. **2**

Normal force on block A due to B and between B and wall will be F.
Friction on A due to B = 20 N
∴ Friction on B due to wall = 100 + 20 = 120 N

*75. Distance of the centre of mass of a solid uniform cone from its vertex is z_0. If the radius of its base is R and its height is h then z_0 is equal to :

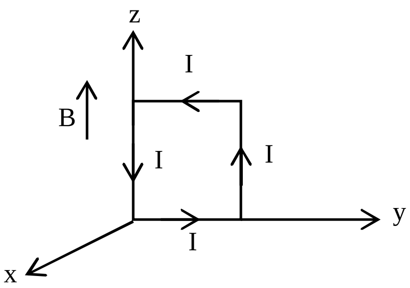
(1) $\frac{3h}{4}$ (2) $\frac{5h}{8}$
(3) $\frac{3h^2}{8R}$ (4) $\frac{h^2}{4R}$

Sol. **1**

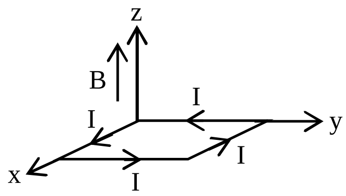
$Z_0 = h - \frac{h}{4} = \frac{3h}{4}$

76. A rectangular loop of sides 10 cm and 5 cm carrying a current I of 12 A is placed in different orientations as shown in the figures below:

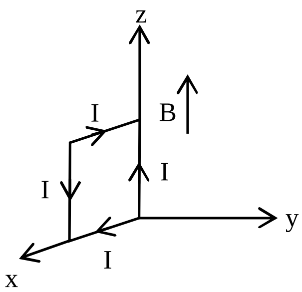
(a)



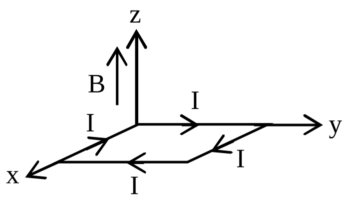
(b)



(c)



(d)



If there is a uniform magnetic field of 0.3 T in the positive z direction , in which orientations the loop would be in (i) stable equilibrium and (ii) unstable equilibrium?

(1) (a) and (c), respectively
(2) (b) and (d), respectively
(3) (b) and (c), respectively
(4) (a) and (b), respectively

Sol. **2**

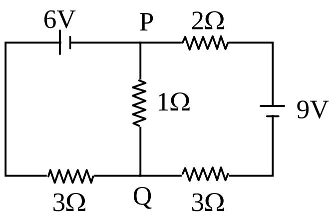
Since $\vec{B}$ is uniform, only torque acts on a current carrying loop. $\vec{\tau} = (I\vec{A}) \times \vec{B}$

$\vec{A} = A\hat{k}$ for (b) and $\vec{A} = -A\hat{k}$ for (d).

$\therefore \quad \vec{\tau} = 0$ for both these cases.

The energy of the loop in the $\vec{B}$ field is : $U = -I\vec{A} \cdot \vec{B}$, which is minimum for (b).

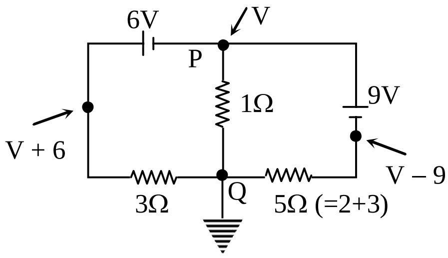
77. In the circuit shown, the current in the 1Ω resistor is :



(1) 0 A
(2) 0.13 A, from Q to P
(3) 0.13 A, from P to Q
(4) 1.3 A, from P to Q

Sol. **2**

Taking the potential at Q to be 0 and at P to be V, we apply Kirchoff's current law at Q:



$$\frac{V+6}{3} + \frac{V}{1} + \frac{V-9}{5} = 0$$

$$V = -\frac{3}{23} = -0.13\,\text{volt}$$

The current will flow from Q to P.

78. A uniformly charged solid sphere of radius R has potential V_0 (measured with respect to ∞) on its surface. For this sphere the equipotential surfaces with potentials $\frac{3V_0}{2}, \frac{5V_0}{4}, \frac{3V_0}{4}$ and $\frac{V_0}{4}$ have radius R_1, R_2, R_3 and R_4 respectively. Then

(1) $R_1 \neq 0$ and $(R_2 - R_1) > (R_4 - R_3)$
(2) $R_1 = 0$ and $R_2 < (R_4 - R_3)$
(3) $2R < R_4$
(4) $R_1 = 0$ and $R_2 > (R_4 - R_3)$

Sol. **2, 3**

The potential at the centre $= k\frac{Q}{\frac{4}{3}\pi R^3}\int_0^R \frac{4\pi r^2 dr}{r} = \frac{3}{2}\frac{kQ}{R} = \frac{3}{2}V_0$; $k = \frac{1}{4\pi\varepsilon_0}$

$\therefore \quad R_1 = 0$

Potential at surface, $V_0 = \frac{kQ}{R}$

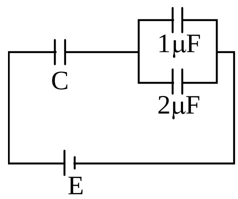
Potential at $R_2 = \frac{5V_0}{4} \Rightarrow R_2 = \frac{R}{\sqrt{2}}$

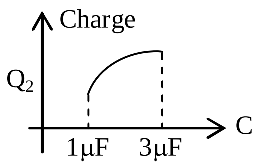
Potential at R_3, $\frac{kQ}{R_3} = \frac{3}{4}\frac{kQ}{R} \Rightarrow R_3 = \frac{4R}{3}$

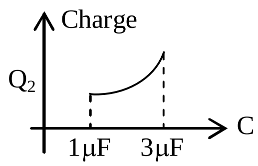
Similarly at R_4, $\frac{kQ}{R_4} = \frac{kQ}{4R} \Rightarrow R_4 = 4R$

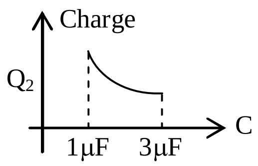
$\therefore$ Both options (2) and (3) are correct.

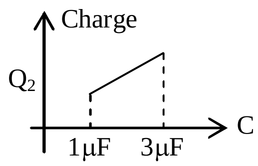
79. In the given circuit, charge Q_2 on the $2\mu F$ capacitor changes as C is varied from $1\mu F$ to $3\mu F$. Q_2 as a function of 'C' is given properly by : *(figures are drawn schematically and are not to scale)*



(1)


(2)


(3)


(4)


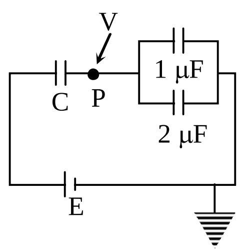
Sol. **1**

Let the potential at P be V,

Then, C (E–V) = 1×V+2×V (we take C in μF)

Or, $V = \frac{CE}{3+C}$

$\therefore \quad Q_2 = \frac{2CE}{3+C}$



*80. A particle of mass m moving in the x direction with speed 2v is hit by another particle of mass 2m moving in the y direction with speed v. If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to

(1) 50% (2) 56%
(3) 62% (4) 44%

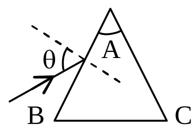
Sol. **2**

$$E_{initial} = \frac{1}{2}m(2v)^2 + \frac{1}{2}2m(v)^2 = 3mv^2$$

$$E_{final} = \frac{1}{2}3m\left(\frac{4}{9}v^2 + \frac{4}{9}v^2\right) = \frac{4}{3}mv^2$$

$$\therefore \quad \text{Fractional loss } = \frac{3-\frac{4}{3}}{3} = \frac{5}{9} = 56\%$$

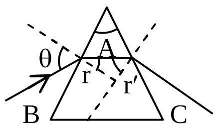
81. Monochromatic light is incident on a glass prism of angle A. If the refractive index of the material of the prism is μ, a ray, incident at an angle θ, on the face AB would get transmitted through the face AC of the prism provided:



(1) $\theta < \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$ (2) $\theta > \cos^{-1}\left[\mu \sin\left(A + \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(3) $\theta < \cos^{-1}\left[\mu \sin\left(A + \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$ (4) $\theta > \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

Sol. **4**



At face AB,

$$\sin\theta = \mu \sin r$$

At face AC $r' < \theta_C$

A-r < $\sin^{-1}\frac{1}{\mu}$

$$\therefore \quad r > A - \sin^{-1}\frac{1}{\mu}$$

$$\therefore \quad \sin r > \sin\left(A - \sin^{-1}\frac{1}{\mu}\right)$$

$$\frac{\sin\theta}{\mu} > \sin\left(A - \sin^{-1}\frac{1}{\mu}\right)$$

$$\theta > \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\frac{1}{\mu}\right)\right]$$

*82. From a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its center and perpendicular to one of its faces is :

(1) $\frac{MR^2}{16\sqrt{2}\pi}$ (2) $\frac{4MR^2}{9\sqrt{3}\pi}$

(3) $\frac{4MR^2}{3\sqrt{3}\pi}$ (4) $\frac{MR^2}{32\sqrt{2}\pi}$

Sol. **2**

For maximum possible volume of cube

$2R = \sqrt{3}a$, a is side of the cube.

Moment of inertia about the required axis = I = $\rho a^3 \dfrac{a^2}{6}$, where $\rho = \dfrac{M}{\frac{4}{3}\pi R^3}$

$$I = \frac{3M}{4\pi R^3}\frac{1}{6}\left(\frac{2R}{\sqrt{3}}\right)^5 = \frac{3M}{4\pi R^3}\frac{1}{6}\frac{32R^5}{9\sqrt{3}} = \frac{4MR^2}{9\sqrt{3}\pi} = \frac{4MR^2}{9\sqrt{3}\pi}$$

83. Match List – I (Fundamental Experiment) with List – II (its conclusion) and select the correct option from the choices given below the list:

List – I		List – II	
(A)	Franck-Hertz Experiment	(i)	Particle nature of light
(B)	Photo-electric experiment	(ii)	Discrete energy levels of atom
(C)	Davison – Germer Experiment	(iii)	Wave nature of electron
		(iv)	Structure of atom

(1) (A) – (ii) (B) – (iv) (C) – (iii)
(2) (A) – (ii) (B) – (i) (C) – (iii)
(3) (A) – (iv) (B) – (iii) (C) – (ii)
(4) (A) – (i) (B) – (iv) (C) – (iii)

Sol. **2**

84. When 5V potential difference is applied across a wire of length 0.1 m, the drift speed of electrons is 2.5×10^{-4} ms^{-1}. If the electron density in the wire is 8×10^{28} m^{-3}, the resistivity of the material is close to:
(1) 1.6×10^{-7} Ωm
(2) 1.6×10^{-6} Ωm
(3) 1.6×10^{-5} Ωm
(4) 1.6×10^{-8} Ωm

Sol. **3**

$J = ne\, v_d$

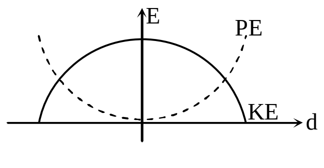
$$\frac{A\Delta V}{\rho \ell A} = nev_d$$

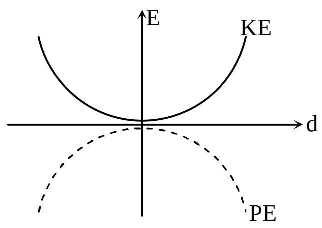
$$\therefore \quad \rho = \frac{\Delta v}{\ell nev_d} = \frac{5}{0.1\times 8\times 10^{28}\times 1.6\times 10^{-19}\times 2.5\times 10^{-4}}$$

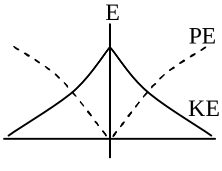
$= 1.56\times 10^{-5}$

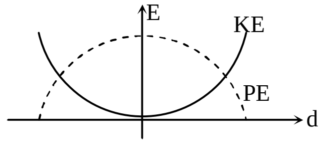
$\approx 1.6\times 10^{-5}$ Ωm

*85. For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly?
(Graphs are schematic and not drawn to scale)

(1)


(2)


(3)


(4)


Sol. **1**

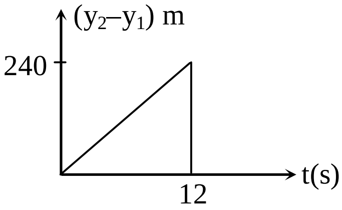
At mean position, K.E. is maximum where as P.E. is minimum

*86. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first ?

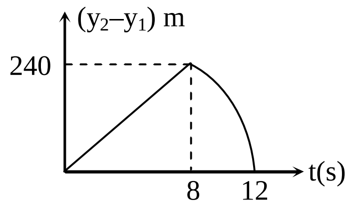
(Assume stones do not rebound after hitting the ground and neglect air resistance, take g = 10 m/s^2)

(*The figures are schematic and not drawn to scale*)

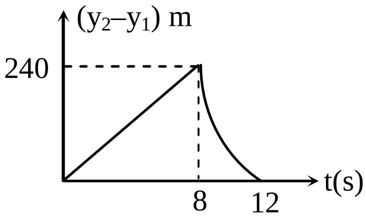
(1)



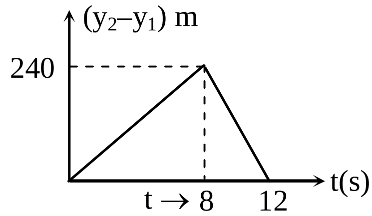
(2)



(3)



(4)



Sol. **2**

$$y_1 = 10t - \frac{1}{2}gt^2$$

$$y_2 = 40t - \frac{1}{2}gt^2$$

$y_2 - y_1 = 30t$ (straight line)

but stone with 10 m/s speed will fall first and the other stone is still in air. Therefore path will become parabolic till other stone reaches ground.

*87. A solid body of constant heat capacity 1 J/°C is being heated by keeping it in contact with reservoirs in two ways:

(i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.

(ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat.

In both the cases body is brought from initial temperature 100°C to final temperature 200°C. Entropy change of the body in the two cases respectively is:

(1) *ln*2, *ln*2 (2) *ln*2, 2*ln*2

(3) 2*ln*2, 8*ln*2 (4) *ln*2, 4*ln*2

Sol. **Correct option is not available**

Case (i) $\int dS = C\left[\int_{373}^{423}\frac{dT}{T} + \int_{423}^{473}\frac{dT}{T}\right] = ln(473/373)$

Case (ii) = $\int dS = C\left[\int_{373}^{385.5}\frac{dT}{T} + \int_{385.5}^{398}\frac{dT}{T} + \int_{398}^{410.5}\frac{dT}{T} + \int_{410.5}^{423}\frac{dT}{T} + \int_{423}^{435.5}\frac{dT}{T} + \int_{435.5}^{448}\frac{dT}{T} + \int_{448}^{460.5}\frac{dT}{T} + \int_{460.5}^{473}\frac{dT}{T}\right] = ln(473/373)$

Note: If given temperatures are in Kelvin then answer will be option (1).

88. Assuming human pupil to have a radius of 0.25 cm and a comfortable viewing distance of 25 cm, the minimum separation between two objects that human eye can resolve at 500 nm wavelength is:

(1) 30 μm (2) 100 μm

(3) 300 μm (4) 1 μm

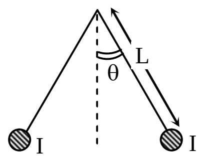
Sol. **1**

$\theta = 1.22\dfrac{\lambda}{D}$

Minimum separation = $(25\times10^{-2})\theta$

= 30 μm

89. Two long current carrying thin wires, both with current I, are held by insulating threads of length L and are in equilibrium as shown in the figure, with threads making an angle 'θ' with the vertical. If wires have mass λ per unit length then the value of I is: (*g = gravitational acceleration*)



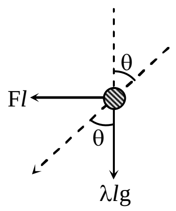
(1) $2\sin\theta\sqrt{\dfrac{\pi\lambda gL}{\mu_0\cos\theta}}$ (2) $2\sqrt{\dfrac{\pi gL}{\mu_0}\tan\theta}$

(3) $\sqrt{\dfrac{\pi\lambda gL}{\mu_0}\tan\theta}$ (4) $\sin\theta\sqrt{\dfrac{\pi\lambda gL}{\mu_0\cos\theta}}$

Sol. **1**

$$\tan\theta = \frac{F\ell}{\lambda\ell g} = \frac{\left(\dfrac{\mu_0 I^2}{4\pi L\sin\theta}\right)\ell}{\lambda\ell g}$$

$$\Rightarrow \quad I = 2\sin\theta\sqrt{\frac{\pi\lambda Lg}{\mu_0\cos\theta}}$$



90. On a hot summer night, the refractive index of air is smallest near the ground and increases with height from the ground. When a light beam is directed horizontally, the Huygens' principle leads us to conclude that as it travels, the light beam:

(1) goes horizontally without any deflection (2) bends downwards
(3) bends upwards (4) becomes narrower

Sol. **3**

According to Huygens' principle, each point on wavefront behaves as a point source of light.